Scene based nonuniformity correction based on block ergodicity for infrared focal plane arrays

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A B S T R A C T
This paper puts forward a new scene based nonuniformity correction algorithm for IRFPA. This method
adopts phase-correlation method for motion estimation and takes the sum of mean-square errors of the
pixel brightness between several adjacent frames as the cost function when the brightness constancy
assumption between two adjacent frames is satisfied. Nonuniformity correction parameters could be
estimated by minimizing such cost function. In order to reduce calculation quantity, we can divide these
images into several subblocks, and solve for the optimum solution of the cost function respectively in each
subblock. From the analysis, it is shown that the optimum solution is of global uniqueness when all the
elements in subblocks could satisfy the ergodicity condition. Then the estimated value of nonuniformity
correction parameters could be deduced by minimizing the cost functions. The nonuniformity correction
experiments for both infrared image sequence with simulated nonuniformity and infrared imagery with
real nonuniformity show that the proposed algorithm could achieve a great correction effect by only
analyzing a small number of frames.

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1. Introduction

With the development of the infrared focal plane array (IRFPA) technology, infrared imaging systems are widely used in such
fields as aviation, industry, agriculture, medicine and scientific researches. However, the response differs in each detector of the
IRFPA and fixed-pattern noise (FPN) is superimposed on the output images, which has greatly degraded the imaging quality, reduced
the spatial resolution and temperature sensitivity [1]. In order to solve this problem, the algorithm for nonuniformity correction
should be applied.

Nonuniformity correction (NUC) techniques can be classified into two categories [2]: calibration-based nonuniformity correc-
tion and scene-based nonuniformity correction. Calibration-based nonuniformity correction algorithms include one-point correction,
two-point correction [3], multi-point correction and so on. But unfortunately, nonuniformity is not immutable and closely related
to external conditions, such as temperature around detectors, bias voltage variation of transistor, change of target irradiation with
time, etc. All these factors would result in slow and random drifting of detector response with time [4], so as to have an influence on
the correction performance of infrared imaging systems. Therefore, these calibration-based NUC methods require the procedure to be
periodically performed so as to guarantee the correction of the temporal drift of the FPN. Thus, scene-based nonuniformity correction
algorithms emerge [5–13] as the times require. Such algorithms, able to self-adaptively correct images with the variation of scene
information, have become a hot spot in nonuniformity correction research.

Currently, the scene-based nonuniformity correction algorithms mostly fall into scene-based statistical methods, such as
temporal high-pass filtering [5], constant statistics method [6], constant range method [7], Kalman filtering method [8], etc. Such
algorithms generally require the relative motion between target scene and IRFPA devices, so that the target scene radiation received
by all the detector units could satisfy a certain statistical constraint. However, such constraints sometimes may not be satisfied
due to the diversity of scenes. Thus, these correction algorithms are frequently accompanied by serious ghosting artifacts [12]. This
problem can be effectively overcome by registration-based nonuniformity correction algorithm, such as O’Neil’s method [9], motion
compensated average [10], algebraic scene-based algorithm [11] etc. However, in this kind of algorithms, computational complexity
and storage demands are relatively high, and the correction errors are easy to transmit accumulatively step by step, which makes
these methods difficult to achieve a practical effect.

A new scene-based infrared nonuniformity correction algorithm is proposed in this paper. In this algorithm, phase-correlation
method is adopted for motion estimation. The irradiation of the same scene between the two adjacent frames is assumed

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unchanged. Compared with brightness constancy assumption that irradiance is constant along the motion trajectory, the assumption presented in this paper could avoid the problem of registration error accumulation. Based on this assumption, the sum of mean-square errors of the pixel brightness between several adjacent frames is taken as the cost functions, which could be minimized to estimate the nonuniformity correction parameters. From the analysis, it can be seen that the optimum solution is of global uniqueness when all the pixels in subblocks could satisfy the ergodicity condition. Then the estimated value of nonuniformity correction parameters could be obtained by minimizing the cost functions.

This paper is organized as follows. Section 2 introduces sensor response model of IRFPA, and points out that drift of IRFPA’s nonuniformity mostly concentrated on the offset. Thus, the proposed algorithm only corrects the offset of nonuniformity and the gain is assumed to be equal to one. Section 3 introduces motion estimation approach based on phase correlation. Section 4 presents the nonuniformity correction algorithm proposed in this paper in detail. Section 5 shows the experimental results. Finally, conclusion is given in Section 6.

2. Sensor response model

First, we assume that the photo-responses of the individual detectors in a focal plane array can be characterized by a linear irradiance–voltage model [13] and their output is given by

$$Y_n(i, j) = g_n(i, j)X_n(i, j) + o_n(i, j)$$  \hspace{1cm} (1)

Here, subscript \(n\) is frame index. \(g_n(i, j)\) and \(o_n(i, j)\) are respectively the real gain and offset of the \((i, j)\)th detector. \(X_n(i, j)\) stands for real incident infrared photon flux collected by the respective detector. In some applications, offset nonuniformity dominates gain nonuniformity; therefore, as a way to simplify the model of Eq.

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**Fig. 1.** An example that meets the ergodicity condition: the shaded part represents the scenes in FOV, one color representing a group of pixels being connected. There is a horizontal displacement of one pixel toward the right side between frame 1 and frame 2, and a vertical displacement of one pixel downward between frame 2 and frame 3.

**Fig. 2.** An example that of the ergodicity condition cannot be satisfied: the shaded part represents the scenes in FOV, one color representing a group of pixels being connected. There is a horizontal displacement of one pixel toward the right side between frame 1 and frame 2, and a vertical displacement of two pixels downward between frame 2 and frame 3.
(1), the gain is assumed uniform across all detectors with a value of unity. Thus the detector model becomes
\[ Y_n(i, j) = X_0(i, j) + b_n(i, j) \]  
(2)

Then nonuniformity correction is given by
\[ X_n(i, j) = Y_n(i, j) - b_n(i, j) \]  
(3)

Therefore, as long as an ideal estimated value of \( b_n(i, j) \) can be acquired, nonuniformity correction could be realized through Eq. (3).

3. Motion estimation

In consideration of two images: \( f_1(x, y) \) and \( f_2(x, y) \), if relative displacements exist respectively in \( x_0 \) and \( y_0 \) in horizontal direction and vertical direction
\[ f_2(x, y) = f_1(x - x_0, y - y_0) \]  
(4)

Based on the Fourier shift theorem
\[ F_2(\omega_x, \omega_y) = F_1(\omega_x, \omega_y) \cdot e^{-j2\pi(\omega_x x_0 + \omega_y y_0)} \]  
(5)

Hence their relative translation can be obtained by calculating their normalized cross power spectrum [14]
\[ \frac{F_1(\omega_x, \omega_y) \cdot F_2^*(\omega_x, \omega_y)}{|F_1(\omega_x, \omega_y) \cdot F_2^*(\omega_x, \omega_y)|} = e^{-j2\pi(\omega_x x_0 + \omega_y y_0)} \]  
(6)

where \([ \cdot ]^*\) denotes complex conjugation, \( F_1(\omega_x, \omega_y) \) and \( F_2(\omega_x, \omega_y) \) are respectively the Fourier transforms of \( f_1(x, y) \) and \( f_2(x, y) \). A Dirac delta function can be gained by inverse Fourier transforming the normalized cross power spectrum. Thus, the relative translation of the two images can be obtained by simply scanning for the global maximum value. The changes of the irradiance mainly have an influence on amplitude while only a minor effect on phase in frequency domain, therefore the phase correlation method is resilient to noise, bad pixels, and other defects typical of infrared images. In this paper, the motion is assumed to consist only of translation, neglecting any scaling, rotation or other warping of the images.

4. Nonuniformity correction

4.1. Brightness constancy equation and cost function

Registration-based nonuniformity correction algorithm is generally required to attach a constraint condition to infrared image sequence, so as to satisfy brightness constancy assumption that the incident radiation of objects along their motion trajectories remain the same. So it is easy to get the satisfying equation between different frames in image sequence.
\[ X_1(i, j) = X_0(i + d_x(n), j + d_y(n)) \]  
(7)

where, subscript \( n = 1, ..., N \) stands for frame index. \( d_x(n) \) and \( d_y(n) \) respectively represent the relative horizontal and vertical shifts between the first frame and \( n \)th frame. Then we get
\[ \sum_{n=2}^{N} \sum_{i,j} [X_n(i + d_x(n), j + d_y(n)) - X_1(i, j)]^2 = 0 \]  
(8)

Due to nonuniformity, the images we observed is \( Y_n(i, j) \) super-imposed nonuniformity offset. Cost function is defined as
\[ C = \sum_{n=2}^{N} \sum_{i,j} [Y_n(i + d_x(n), j + d_y(n)) - Y_1(i, j)]^2 \]  
(9)

It is impractical to estimate nonuniformity offset by minimizing the cost function in Eq. (9), because motion estimation is generally performed between adjacent frames. Thus, the backer the frame index involved in the operation is, the larger the error of the relative translation estimate will be. The accumulative errors would make it difficult to accurately estimate \( d_x(n) \) and \( d_y(n) \) in Eq. (9), which would result in significant error of \( C \). In addition, large computational load to solve for the cost function of Eq. (9) makes it unsuitable for real-time applications. Finally, we have no idea what is the least numerical value taken for \( N \) can ideal nonuniformity offset estimate be acquired by minimizing the cost function. In the following sections, we will conduct a further analysis on these problems.

4.2. Modified cost function

If the scene motion trajectory could be correctly obtained, and there is no mutation occurring to the temperature field of each object in the field of view (FOV), constraint condition of Eq. (7) would be satisfied easily. But these limits appear too strict in the practical application. Thus, we hereby relax this constraint condition as: the irradiance of objects does not change during the iframe time [15].
\[ X_n(i, j) = X_{n+1}(i + d_x(n+1), j + d_y(n+1)) \]  
(10)

where, subscript \( n = 1, ..., N - 1 \) stands for frame numbers. \( d_x(n + 1) \) and \( d_y(n + 1) \) respectively represent the relative horizontal displacement and vertical displacement of frame \( n \) and frame \( n + 1 \). Then Eq. (8) could be modified as
\[ \sum_{n=1}^{N-1} \sum_{i,j} [X_n(i + d_x(n+1), j + d_y(n+1)) - X_1(i, j)]^2 = 0 \]  
(11)

Accordingly, Eq. (9) could be modified as
\[ C = \sum_{n=1}^{N-1} \sum_{i,j} [Y_{n+1}(i + d_x(n+1), j + d_y(n+1)) - Y_1(i, j)]^2 \]  
(12)

It is not hard to find out: substituting Eq. (12) for the cost function of Eq. (9) could avoid the problem of registration error accumulation. In addition, this constraint condition would be easier to be satisfied because it only requires brightness constancy between two adjacent frames.

4.3. Block algorithm and uniqueness of solution

As is referred above, to find the solution of the cost function of (12) is also not practical, so we divide the image into several non-overlapping subblocks, and minimize the cost function of (12) in each subblocks independently.
\[ C_B = \sum_{n=1}^{N-1} \sum_{i,j} [X_{n+1}(i + d_x(n+1), j + d_y(n+1)) - Y_1(i, j)]^2 \]  
(13)

Without loss of generality, we suppose that the average of the nonuniformity offset parameters in each block to be zero.
\[ \sum_{i,j} b(i, j) = 0 \]  
(14)

Among which, \( M_B \) and \( N_B \) stands for the number of rows and columns of each block. And then put Eq. (2) into Eq. (13) as substitution.
\[ C_B = \sum_{n=1}^{N-1} \sum_{i,j} [X_{n+1}(i + d_x(n+1), j + d_y(n+1)) + X_0(i, j) - b_n(i+1, j+1) - b_n(i, j)]^2 \]  
(15)
Therefore, we can get a quadratic optimization problem with a linear constraint:

\[
\begin{align*}
\min f(x) &= \frac{1}{2} x^T Q x + q^T x \\
\text{s.t. } a^T x &= 0
\end{align*}
\] (16)

In the equation, the vector \( x^T = [b_1, b_2, \ldots, b_{M_B \times N_B}] \) sequence contains nonuniformity offset of \( M_B \times N_B \) elements in the block. The symmetrical matrix \( Q_{M_B \times N_B} \) is the Hessian matrix of the quadratic coefficient in the Eq. (15). Through the features of Eq. (15), it is not difficult to find out that the Hessian matrix \( Q \) is a positive semidefinite matrix, and the equation (16) at the time is a convex programming problem. If it meets the quadratic sufficient conditions, viz

\[
d^T Q d > 0 (\forall d \in \mathbb{R}^n, d \neq 0), \quad a^T d = 0
\] (17)

This convex programming problem has a unique global optimal solution. And it can be transferred into the following linear equations to obtain the solution [16]

\[
\begin{pmatrix}
4 & -2 & -2 \\
-2 & 6 & -2 \\
-2 & -2 & 6
\end{pmatrix}
\begin{pmatrix}
x \\
-2 & 8 & -2
\end{pmatrix}
= 
\begin{pmatrix}
2 & -2 & 8 & -2 \\
-2 & 8 & 2 & -2
\end{pmatrix}
\]

\[
\begin{pmatrix}
Q & -a \\
a & 0
\end{pmatrix}
\begin{pmatrix}
x \\
\lambda
\end{pmatrix}
= 
\begin{pmatrix}
q \\
\lambda
\end{pmatrix}
\] (18)

The coefficient matrix in Eq. (18) \( \begin{pmatrix} Q & -a \\ a & 0 \end{pmatrix} \) is Lagrange matrix. It is obvious that when \( Q \) meets the quadratic sufficient condition, the rank of Lagrange matrix \( R \begin{pmatrix} Q & -a \\ a & 0 \end{pmatrix} = M_B \times N_B + 1 \), the group equations of Eq. (18) has the unique solution.

4.4. Ergodicity of subblock

From Section 4.3, we know the necessary and sufficient condition for the cost function Eq. (13) to have a unique global optimal solution. But we probably do not know that how much should \( N \) be at least will this condition be satisfied. In this section, we will give a more direct equivalent condition, which is called ergodicity condition. When the same scene appears at two different pixels of two frames, the two pixels can be called being “connected”. When all the pixels in the subblocks are connected, it can be called satisfying the ergodicity condition, and at this time Eq. (16) has a unique global optimal solution. So here let’s ignore the strict mathematical confirmation, and illustrate it with a very simple example:

The three \( 5 \times 5 \) grids in Fig. 1 represent the movement in successive three frames of a \( 5 \times 5 \) subblock in an image, the shaded part represents the scenes in FOV, one color representing a group of pixels being connected. There is a horizontal displacement of one pixel toward the right side of the second frame to the first frame, and there is a vertical displacement of one pixel downward of the third frame to the second frame. If the color of the grids is the same, then it indicates that they are connected. As what we can see, all the 25 pixels are connected in the whole block after the shifts of two frames. Let us look at the sub matrix \( Q_{11} \) in the corresponding Lagrange matrix:

\[
\begin{pmatrix}
4 & -2 & -2 & -2 & -2 \\
-2 & 6 & -2 & -2 & -2 \\
-2 & -2 & 6 & -2 & -2 \\
-2 & -2 & -2 & 6 & -2 \\
-2 & -2 & -2 & -2 & 6
\end{pmatrix}
\]

It can be obtained that \( R \begin{pmatrix} Q & -a \\ a & 0 \end{pmatrix} = M_B \times N_B + 1 \). Obviously, the quadratic optimization problem of Eq. (16) has a unique global optimal solution.

Let us look at another condition under which there is a horizontal displacement of one pixel toward the right side of the second frame to the first one, and there is a vertical displacement of two pixels downward of the third frame to the second one in Fig. 2. We can see that, after the movement of the two frames, the twenty five pixels in the whole block are not all connected. They are split into two groups: one group is made up of the first, the third, and the fifth rows; and other made up of the second and the fourth
rows. All the elements in the group are connected but not between the groups. Then let us look at the subblock $Q_2$ corresponding to Lagrange matrix:

$$
\begin{pmatrix}
4 & -2 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
-2 & 4 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
-2 & 4 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
\end{pmatrix}
$$

For exchange of some two rows or two columns of matrix cannot change the rank of the matrix, so after we exchange some rows and columns of $Q_2$, can be obtained as the following:

$$
\begin{pmatrix}
4 & -2 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
-2 & 4 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
-2 & 4 & -2 & -2 \\
-2 & 6 & -2 & -2 \\
\end{pmatrix}
$$
Fig. 3. Simulated nonuniformity image correction results. (Frame 4) (a) image with simulated nonuniformity (RMSE = 50), (b) uncorrupted image, (c) estimated offset, (d) corrected with proposed method (RMSE = 6.92).

Fig. 4. Results of nonuniformity correction of the real data. (a) Image with nonuniformity, (b) corrected image (without DC offset correction), (c) estimated offset nonuniformity, (d) corrected image (with DC offset correction).
It can be found out that \( Q_x \) can be divided into four blocks, and the two subblocks that are not 0 represent the parameter corresponding to two groups of pixels that are not connected. And the determinants of two subblocks are both 0, so \( R \left( \begin{array}{cc} Q_x & -a \\ a & 0 \end{array} \right) < M_B \times N_B + 1 \), and thus the condition that the quadratic optimization problem of Eq. (16) has a unique global optimal solution is not satisfied.

From the examples above, we can know that if the pixels in the subblocks meet the ergodicity condition, that the quadratic optimization problem of Eq. (16) has a unique global optimal solution. We can obtain solution of nonuniformity correction parameters by solving the linear equations of Eq. (18). If the scene is in random motion, the ergodicity condition can be satisfied with only a few blocks. And it is easy to find out that when one of the blocks in the image meets the ergodicity condition, and so are the other blocks.

As to the judgment to the ergodicity, the way of adjacent matrix can be applied [17], whose definition is as follows

\[
A = \begin{cases} \begin{array}{cc} (M_B M_B + M_B N_B) & 1 \\ \times \times \end{array} & \begin{array}{cc} M_B N_B & 0 \end{array} \\ \begin{array}{cc} 1 & 0 \end{array} \\ \begin{array}{cc} 0 & 1 \end{array} \end{cases} \quad \begin{array}{c} \text{when the } i^{th} \text{ and } j^{th} \text{ pixels are connected} \\ \text{otherwise} \end{array} \quad (19)
\]

When the adjacent matrix meet the condition that all the elements in the matrix \( S \) of Eq. (20) are not 0, it indicates that all the pixels in the block are connected.

\[
S = \sum_{k=1}^{M_B N_B} A^k
\]

Another equivalent way is to judge the singularity of the Lagrange matrix, when the determinant is not 0, the only solution of the nonuniformity correction parameter can be solved by solving the linear equations of Eq. (18).

4.5. Treatment of the block effect

The above discussed nonuniformity correction is performed independently in each subblock. And we assume that the average of nonuniformity offset parameters in the subblocks is 0. This assumption is sometimes not reasonable between blocks, because the direct current (DC) component of the nonuniformity offsets in each block cannot be guaranteed equal. So the block effect will appear in the corrected image inevitably. In order to solve this problem, we can overlap some pixels of rows and columns between the adjacent blocks while dividing the image into blocks. The DC component of the offset between the adjacent blocks can be corrected through these overlapped pixels. In this way, the block effect can be eased.

5. Experimental result

In this section, the nonuniformity correction algorithm proposed in this paper is tested with infrared data corrupted with simulated nonuniformity first. Then a 320 × 256 mid-wave cooled infrared camera is used to test the proposed algorithm under the real nonuniformity condition. The root-mean-square error (RMSE) can be adopted for measurement to have an objective evaluation of the correction performance of a certain NUC algorithm, which is defined as follows

\[
\text{RMSE} = \sqrt{\frac{1}{M_X \cdot N_X} \sum_{i,j} (X(i,j) - \hat{X}(i,j))^2}
\]

Where \( X(i,j) \) is the (i, j)th pixel's value of the true frame while \( \hat{X}(i,j) \) is the pixel's value of the corrected frame. \( M_X \) and \( N_X \) are respectively the rows and columns of the image. The first infrared sequence with artificial nonuniformity is generated from a clear infrared video sequence, using a synthetic offset with a zero-mean Gaussian distribution with standard deviation of 50. The effective of the proposed approach is illustrated in Fig. 3. Fig. 3(a) and (b) shows the 4th corrupted image and its clean version. Fig. 3(d) shows the corresponding corrected one. The estimated correction offset coefficients are given in Fig. 3(c). From Fig. 3, it can be seen that the proposed algorithm only requires a small number of samples to exhibit an excellent reduction of FPN. Because the stimulated nonuniformity is roughly evenly distributed in each block, there is no need to correct the DC offset. After correction by the proposed algorithm, the RMSE is reduced from 50 to 6.92.

Next, the algorithm put forward is applied to an uncorrected video sequence acquired from a 320 × 256 mid-wave IRFPA. Its original image, corrected image and estimated offset nonuniformity referring to Fig. 4(a)–(c). The block size is 32 × 40. The ergodicity condition is met after 5 frames of scene movement, and then the correction is performed. Serious striping effect can be found in the Fig. 4(a) and it is probably caused by the nonuniformity in the readout electronics. After the nonuniformity correction through the proposed algorithm, the image appears to show significant improvement and the nonuniformity is reduced a lot. However, as what we discussed before, the block effect still exists in the corrected image. After revising the subblock size to 35 × 43 through overlapping 3 pixels in both row and column and corrected the DC component of offset in each block, we can see from Fig. 3(d) that block effect has been eliminated effectively and the image shows a more desirable observation result.

6. Conclusion

This paper described a new scene based nonuniformity correction algorithm for IRFPA. In this algorithm, we assume the irradiance of objects does not change during the time between image frames, so that the total brightness mean square error of several adjacent image pixels can be minimized and the nonuniformity correction parameters of the IRFPA can be estimated.

We have mainly discussed three problems: First, we talk about the choice for cost function. We adopt the total brightness mean square error of several adjacent image pixels to avoid registration error accumulation and make full use of all the scene information. Second, we come to reducing amount of calculation. We divide the image into subblocks and estimate the nonuniformity parameter independently to reduce the amount of calculation effectively. Third, we talk about how many frames we should take to optimize the equation. We hold that when the pixels of each subblock meet the ergodicity condition, the optimization problem has a unique solution. We also prove that the ergodicity condition can be met through only a few frames by examples. In addition, we analyze the problems resulting from dividing the image into blocks, and put forward an easy and feasible method to deal with the block effect.

The performance of the proposed algorithm is evaluated with both infrared image sequence with simulated nonuniformity and infrared imagery with real nonuniformity. The result indicates that the algorithm can achieve a favorable nonuniformity correction effect through only a few frames and proves to be a feasible and effective scene based nonuniformity correction method.

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