# Light field moment imaging: comment

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We comment on the recent Letter [Opt. Lett. **38**, 2666 (2013)], in which the authors presented an imaging technique called light field moment imaging. We wish to show that this method can be associated with transport of intensity equation at the geometric optics limit. 

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In a recent Letter, Orth and Crozier [1] reported a method termed light field moment imaging (LMI) to extract the first angular moment of the light field by solving a partially differential equation (PDE) with only a pair of images exhibiting a slight defocusing. With the moment retrieved, one can readily reconstruct perspective-shifted views of the original scene under the empirical assumption that the angular distribution of the light field is Gaussian. The main purpose of this Letter is not to discuss the reasonability or to improve this empirical assumption. In fact, we want to establish the relation between LMI and the deterministic phase retrieval method—transport of intensity equation (TIE) [2]. Starting with the more rigorous wave optics, we represent the paraxial quasi-monochromatic partially coherent stochastic field $u(\mathbf{x})$ using Wigner distribution function (WDF) [3]:

$$W(\mathbf{x},\mathbf{u}) = \int \Gamma(\mathbf{x}+\mathbf{x}'/2, \mathbf{x}-\mathbf{x}'/2)\exp(-i2\pi\mathbf{u}\mathbf{x}')\mathrm{d}\mathbf{x}', \quad (1)$$

where $\mathbf{x}$ and $\mathbf{u}$ are the two-dimensional (2D) spatial and spatial frequency vectors and $\Gamma$ is the mutual intensity. The paraxial propagation of WDF takes the form $W(\mathbf{x},\mathbf{u},z) = W(\mathbf{x}-\lambda z\mathbf{u},\mathbf{u},0)$, where $z$ is the propagation distance [3]. Combining the representation of intensity—a projection of the WDF onto $\mathbf{u}$ plane $I(\mathbf{x}) = \int W(\mathbf{x},\mathbf{u})\mathrm{d}\mathbf{u}$, we obtain the TIE for the partially coherent field [4]:

$$\frac{\partial I(\mathbf{x})}{\partial z} = -\lambda\nabla_{\mathbf{x}}\cdot\int \mathbf{u}W(\mathbf{x},\mathbf{u})\mathrm{d}\mathbf{u}, \quad (2)$$

where $\nabla_{\mathbf{x}}$ is the gradient operator over $\mathbf{x}$. In the coherent limit $u(\mathbf{x}) = \sqrt{I(\mathbf{x})}\exp[i\phi(\mathbf{x})]$, Eq. (2) reduces to Teague's TIE [2], and the normalized spatial frequency moment of WDF relates to the phase gradient:

$$\frac{\int \mathbf{u}W(\mathbf{x},\mathbf{u})\mathrm{d}\mathbf{u}}{\int W(\mathbf{x},\mathbf{u})\mathrm{d}\mathbf{u}} = \frac{1}{2\pi}\nabla_{\mathbf{x}}\phi. \quad (3)$$

The formal resemblance between TIE [Eq. (2)] and the PDE {Eq. (3) in [1]} of LMI implies the equivalence of the WDF and the light field; that is, the physically measurable light field $L(\mathbf{x},\boldsymbol{\theta})$ approaches WDF at geometric optics limit [5]. Applying the approximation $L(\mathbf{x},\boldsymbol{\theta}) \approx W(\mathbf{x},\lambda\mathbf{u})$ to Eq. (3),

$$\frac{\partial I(\mathbf{x})}{\partial z} \approx -\nabla_{\mathbf{x}}\cdot\int \boldsymbol{\theta}L(\mathbf{x},\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta}. \quad (4)$$

Invoking the paraxial approximation $\boldsymbol{\theta} \approx (\tan\theta_x, \tan\theta_y)$ to eliminate the illumination factor $\cos^4\theta$ [1], the "recorded light field" reduces to the $L(\mathbf{x},\boldsymbol{\theta})$ and then Eq. (4) reduces to Eq. (3) in [1]. Therefore, the PDE of LMI is equal to partially coherent TIE at the geometric optics limit. It is seen from Eq. (3) that the "phase" measured by TIE is a scalar potential whose gradient yields the (irrotational component of) normalized first-order local moment of WDF, which describes exactly the normalized ensemble-averaged transverse energy flux density (Poynting vector). Under the geometric optics assumption, the Poynting vector propagates in the Eikonal way; thus, the ensemble-averaged transverse Poynting vector is equal to the angular moment of the light field in LMI.

For a partially coherent field, the 4D WDF is generally non-redundant, and two projections of WDF are insufficient to fully recover the field. The empirical Gaussian angular distribution assumption in [1] seems not physically founded, but it raises an interesting question: is there any other situation (except the coherent case) that we can fully characterize the optical field without measuring the whole phase–space distribution? The answer, apparently, is yes. However, this question is not trivial, but of great practical importance, because identifying the phase–space redundancies explicitly enables the derivation of more-efficient computational schemes for specific problems. We leave this for future investigations.

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